

Symmetry Breaking unifies Forces

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ABSTRACT

In physics an open question is how general relativity can be fitted to the standard model of physics. The author shows how this can be done by using octonians, catastrophes for symmetry breaking, complex cross ratios as six valued color charge force and inner dynamics for fermions. Essential is a projective geometrical view, extending the affine from special relativity.

Higgs and the Standard Model of Physics : The finding of Higgs bosons ended in a first round the question whether or not the Standard Model of Physics SMP has to be given up. It has the symmetry $U(1) \times SU(2) \times SU(3)$. The finding of a Higgs field, attributing mass to systems in the universe by using Higgs bosons as field quantum is only the first step in saving SMP. General relativity is still not compatible with it. For Higgs bosons is quoted that the shape of the polynomial $y = x^4$ is rotated, generating a pigtrough kind of 2-dimensional surface (figure 1). This polynomial has to be replaced through a catastrophe.

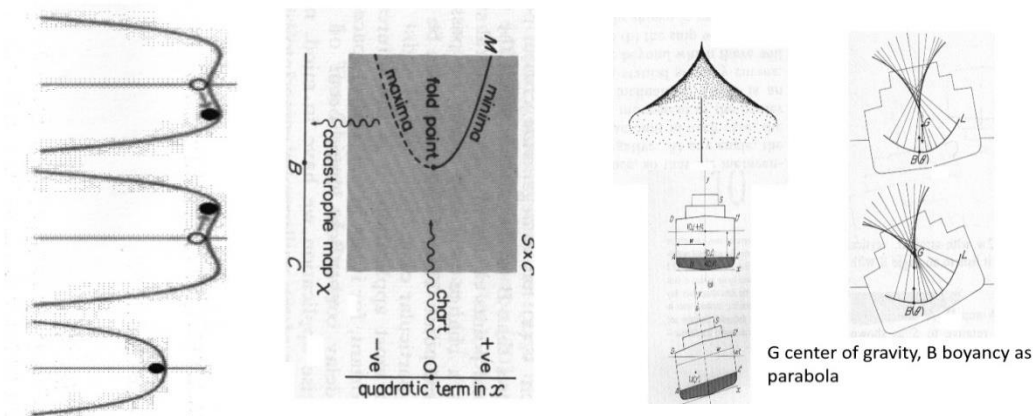


Figure 1 left, middle and right fold parabola $y = x^2$ and ship boyancy $B(\theta)$ with the extended equation $y = (V_a) = x^3 + ax$ third row top and forth row upper parts on ship; first row two symmetry broken pigtroughs where a parabola point $y = 0$ is replaced by two possible (black marked) points at $y = +1/2$ and $y = -1/2$, the new cusp catastrophe is added in upper parts left

The ship example is also seen for other gravitational (Zeeman machine) bound elastic structures, having for the butterfly catastrophe a pigtrough occurring. The actual equilibrium positions for the ships barycenter G of a boyancing ship changes the fold catastrophe (used as $y = x^2$ ($a = 0$)) or cubic as V_a $a \neq 0$) to the catastrophe map $y = (x^2 + y^2)^2 - a(x^2 + y^2)$, $a > 0$, $b = 0$, as stream function of a *cusp* potential $V_{ab} = x^4 + ax^2 + bx$, a, b parameters. The case $b = 0$ is not structurally stable; this is obtained for V by adding a nonzero perturbation term $b \neq 0$ for the cusp which is structurally stable (meaning that small perturbations of parameters don't give sudden changes of states, for instance for the a ship falling to its right side, not showing a pendulum motion towards the water surface). For the graph of the V_{ab} normed derivative $x^3 + ax + b = 0$ the graph for $b = 0$ is a parabola with the line $x = 0$ (fold), for $b \neq 0$ the graph is disconnected, differentiating y for the pigtrough gives a curve with an inflection point and a parabola shaped disjoint curve (see figure 2). For the ship boyancy cusp, there is a superposition of two hyperbolic umbilic catastrophes with potentials $V_{abc} = x^2y + y^3 + ax^2 + by + cx$. In superposition of four hyperbolic umbilics they show also up for drawings with the Zeeman gravitational machine where the force is transferred by elastic rubber bands which can move a central disk fixed on a line along cusp lines and as sudden changes, jumps of the disk at cusp points (figure 3) occur.

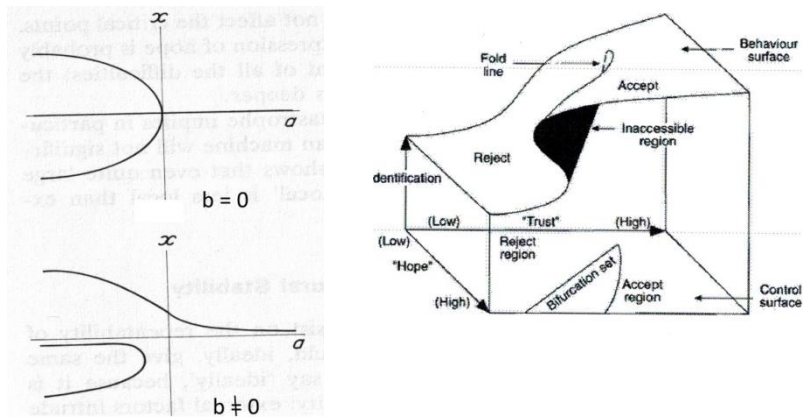


Figure 2 parameters a,b for fold and cusp; a cusp drawn 3-dimensional

For catastrophes are introduced parameters like a (or b), not new variables like θ as a spherical space angle, leaning the ship in an angle towards the water surface xy -plane E as vertical line, and measured towards z -axis as normal space direction to E . Mentioned is briefly, that for real spaces of a local manifold as R^n , here $n = 2$ for surfaces, the parameters are not increasing the dimension and are replaced by a smooth map $g: R^n \times R^k \rightarrow R^k$ such that for a family of R^n diffeomorphisms $f_s, s \in R^k$, the structure of a flow is kept, but stretched or squeezed in a flat, transversal presentation while a diffeomorphism $e: R^k \rightarrow R^k$ keeps the shape of a 3-dimensional figure (deform for instance a torus of genus 1 and fix it a solid stick); for parameters there is a smooth map $\gamma: R^k \rightarrow R$. They give an equivalent map to g as $h(x,s) = g(f_s(x), e(s)) + \gamma(s)$ in a (x,s) -neighborhood of 0. In the fold case, $k=1$, for the cusp $k=2$. Manifolds with the critical points Morse lemma are using only one f , no index is necessary.

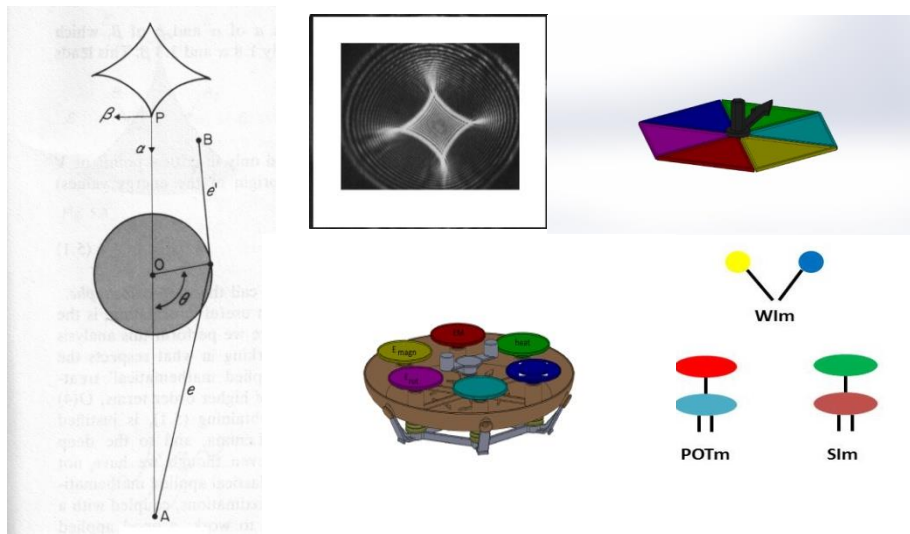


Figure 3 Zeeman machine [3], the four cusps with LASER drawn; below 3 motors, at right G-compass and 6 roll mill

Another very useful catastrophe for nucleons is

the elliptic umbilic (D_4^-)

$$u_1^2 u_2 - u_2^3 + t_3 u_1^2 + t_2 u_2 + t_1 u_1 + (N);$$

it adds as potential $V_{abc} = x^2 y - y^3 + ax^2 + by + cx$ with two speeds for the inner dynamics of a nucleon. A gluon-quark flow is driven by three motors POT, SI, WI (figure 3) where WI (weak interaction) runs with a different speed than SI (strong interaction) and POT (inner electrical plus gravity potentials) has the same speed as SI.

There are sudden state changes of the nucleon when two quarks exchange a gluon [1]. In [5] is as model the 6 roll mill for this, each quark motor drives two color charged rolls for the inner gluon-quark flow. Symmetry breaking with the pigtrough of Higgs includes as sudden change that a Higgs boson decays. This is also assumed for a dark matter Q decay, observed for the big bang. Quarks and leptons are generated. As cusp catastrophe application for Q is assumed that the quasiparticle wrinkleton makes on the aggreption disk a cusp folding. The structural instability of Q can be that it has as integrated catastrophe one parameter more which is set 0, keeping the cusp (as derivative) parameters $\neq 0$. The decay products split in a cusps parameters control space into two parts which have the maximum or minimum potential of the cusps V as a bifurcation. The bifurcation is from the POT motor for electrical charge and mass potentials energies and quarks as POT field-like quantum which are presented as 2 roll mill (figure 4) driven by the POT motor and having these two energies as poles for the two rolls. In a retract version, a lemniscate in the inner flow of a quark as brezel of genus 2 is obtained. This is a Lissajous figure when two frequencies hit orthogonal in proportion 1:2 or 2:1. For nucleons as brezels of genus 3 such Lissajous figures use the proportions 1:3. In a crystalized version is drawn a tetrahedron where at the tip of the tetrahedron sits a *rgb*-graviton as spin-like base GF triple in xyz-space and neutral superposition of three quarks with their barycenters added at the ends of the *rgb*-graviton. This can be called a measuring kg GF while the spin GF $s = (s_x, s_y, s_z)$ is measuring length. In contrary to spin, the x,y or z kg vectors can carry 2 or 3 different weights. In a nucleon ddu or uud the d-, u-quarks have different weights. The *rgb*-graviton has no weight and spin 2, the quarks have for measuring length a spin $\frac{1}{2}$ attached, gluons as bosons have integer spin 1.

Quarks and weak boson decay : After a Higgs decay, generated quarks are also decaying. They get structural stable only in a nucleon superposition, attached to a *rgb*-graviton, using their gluon exchange for confinement in the nucleon and the dynamics of the strong SI rotor [6]. The elliptic catastrophe serves for nucleons.

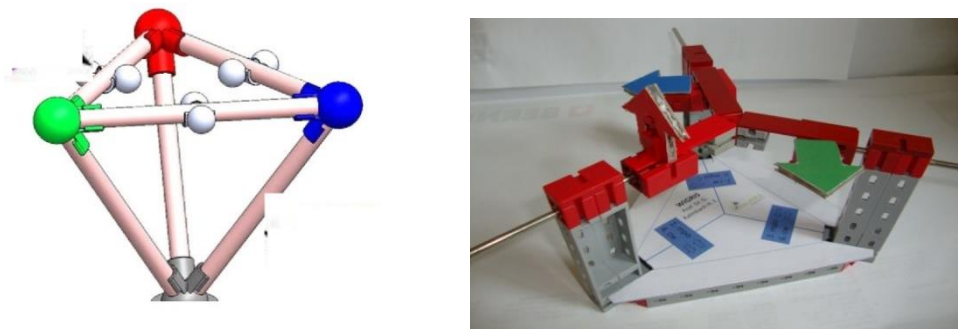


Figure 4 nucleon tetrahedron with a *rgb*-graviton GF having at its vectors ends the quarks color charges red, green, blue as vertices; on sides of the triangle the two small balls are for a gluon exchange on this side; nucleons SI rotor

The quark decay for a u- (or d-)quark is with a weak boson W^+ (or W^-) which itself is decaying according to the Feynman diagrams into leptons, for instance a positron and neutrino (or electron and antineutrino). A u- (d-)quark becomes then a d- (u-)quark.

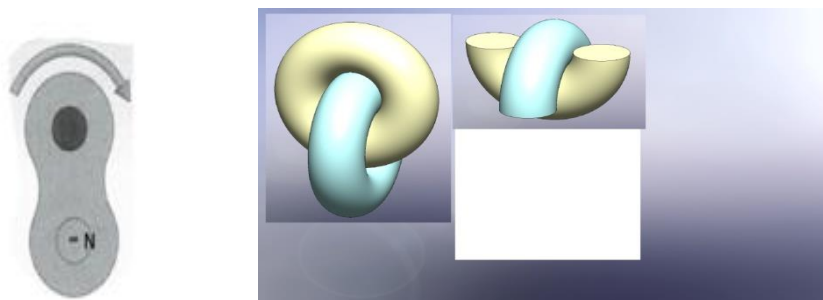


Figure 5 quark (a 3-dimensional solid brezel of genus 2) with two poles for its electrical and kg charges and with an inner rotation for its 2 roll mill flow (in figure 6 part (e)); Heegard decomposition of the weak bosons sphere S^3 into two lepton (genus 1) tori with a decay figure at right, the parts close again to two tori for electron and antinautrino or positron and neutrino



Figure 6 electron torus as base, neutrino toroidal spindle as base; attached are 4-dimensional coordinates of xyzt-space, not a 3-dimensional *rgb*-graviton as for nucleons; the spindle can be closed to a pinched torus where one transversal torus circle is projective retracted to a point ∞

It was assumed that the quasiparticle wrinkleton acts for the Higgs decay. The quark decay is observed when two protons get a central common barycenter set for its mass with a Higgs boson (figure 7). In the position at left two u-quarks are oppositely located on a line. The upper one is decaying and becomes in the right figure a d-quark. This allows after the rotation of the upper tetrahedron (where in fusion a positron and antineutrino is emitted) in the environment a newly generated inner deuteron weak WI rotor. The nucleons quarks are paired in deuteron on x-, y- or z-axis as ud and get a weak isospin exchange, replacing the strong gluon exchange between quarks. The nucleons exchange their states, the location of the proton, neutron parts where the attached positron of the proton also moves. In atomic kernels the central point of the double tetrahedron can be bifurcated and an exciton keeps them in distance together with the isospin exchange.

Symmetry braking : As for Higgs, the quark and weak boson decays can be called symmetry braking. The description is through catastrophes which allow these sudden changes. A projective 5-dimensional POT field is described in [8]. It has a rolled Kaluza-Klein coordinate and octonian coordinates, listed by indices, 123456. They are for color charges with coordinates, an energy and a symmetry attached. The new symmetry is D_3 of order 6 for a SI rotor (changing states), discrete as permutations of three elements r,g,b. In gluon exchanges, the quarks can change their color r,g or b. Numerical they stand for the projective numbers 0,1, ∞ as reference points on a real $U(1)$ or complex rolled S^2 projective line. Adding a complex number z for permutations of 4 elements with the S_4 symmetry, there are six complex cross ratios with the elements of D_3 , a color charge, a coordinate 1,2,... and an energy attached. The 4-tuples are 1 x, red r, EM(pot) and id, the identity symmetry; 2 y or polar φ , green g, E(heat) for temperature and $\alpha\sigma_1$ a reflection, σ_1 the first Pauli matrix, α a 120° covering rotation of

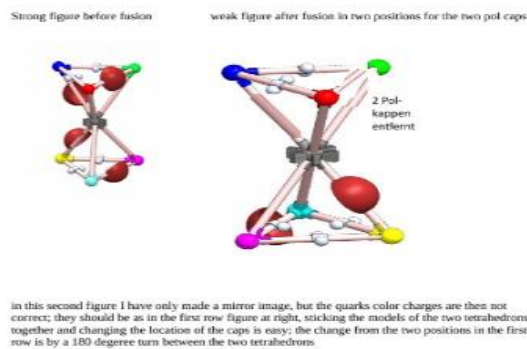


Figure 7 fusion of two protons to deuteron with one proton and one neutron as nucleon states

the quark triangle; 3 z or spherical angle θ , c(g) magenta, rotational energy $E(\text{rot})$ with angular momentum $L = r \times p$, $p = mv$ momentum, r radius, and α^2 ; 4 ct time, c speed of light, c(b) yellow, $E(\text{magn})$ magnetic field strength and $\alpha^2\sigma_1$; 5 mass, turquoise c(r) and $E(\text{pot})$ gravity potential, σ_1 ; 6 frequency $f = 1/\Delta t$ as inverse time interval, blue b, $E(\text{kin})$ kinetic energy with momentum p, α as symmetry. Each symmetry provides a force eigenvector for the measure of the energy such as meter for length (on 1), kg for mass (on 5), second for time (on 4), Hz for f (on 6), Kelvin for heat (on 2),... The vector points in direction of the energy coordinate; the

color charge is presented as the scaled coefficient matrix of the symmetry written as complex cross ratio (Moebius transformation); $c(u)$ is the dual color charge of $u = r, g, b$. The duals occur through an application of the conjugation operator of physics in the Heisenberg uncertainties coupling where $u, c(u)$ are on one space coordinate in opposite direction, x for $u = r$, y for $u = g$ and z for $u = b$ (figure 8). The inner SI rotor is a symmetry breaking by using the elliptic catastrophe. The tetrahedron symmetry is S^4 not belonging to SMP. It is factorized by the normal CPT Klein group of order 4 for the listed strong SI factor classes. The weak WI isospin exchange is not SMP and uses another rotor [1]. Catastrophes related to weak decays are for nucleons the elliptic umbilic, for quarks can serve the 2 roll mill representation (figure 9 (e) flow) which has the equation $V_{ab} = ax^2 + by^2$, $a/b < 0$ as stability of the Morse saddle.

In the projective plane 56 the equation $mc^2 = hf$ allows to change mass in frequency and reversely. For frequency there are three options: f as inverse time interval, $\omega = 2\pi f = d\phi/dt$ angular, rotational speed and $f = n$, counting winding numbers about a circle S^1 . This is computed through complex residual contour S^1 integration $n = a \oint dz/z$. Geometrical it is for a real line as octonian coordinate 7, rolled on a cylinder with a transversal circle as section S^1 and it is rolled on the cylinder surface as helix line where the wave length in an exponential Ψ wave presentation is counted by the natural number n . This is another SMP symmetry braking. As new matrix of order 2 can be used the scaled 2x2-matrix of the Minkowski metrics scaling factor, in D_3 belonging to time 4 as a translation. The crossed pigtrough presentation belongs to Lissajous figures. The Ψ wave cylinders of 7 for two hitting frequencies are crossing in a xy -plane with their transversal circle in xz - or yz -direction. The generated Lissajous figure, a circle (in octonians 7 as $U(1)$ for electromagnetic waves with its universal covering) for the 1:1 proportion, is in the xy -plane. Repeated is that the proportions 1:2 (1:3) give quarks (nucleons) as genus 2 (3) surfaces while 1:1 makes a torus surface, all bounding a solid 3-dimensional space for the systems.

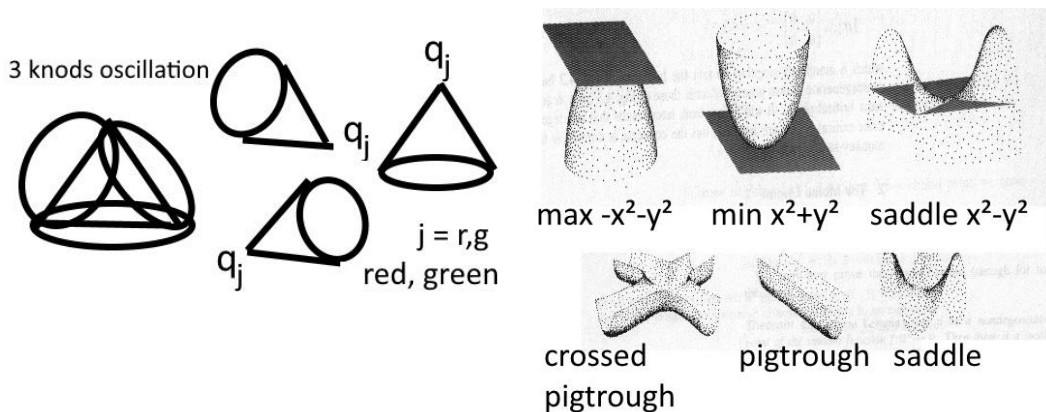


Figure 8 harmonic oscillation with cones and triangle for the nucleons SI rotor; three Morse critical points for maximum of V , minimum or saddle with the crossed pigtrough presentation

In figure 8 at left is shown how the SI rotor makes a standing oscillation about the quark triangle sides: one vertex red or green is kept fixed and a barycentric coordinate through this point P and the midpoint of the opposite side is a cones rotation let the triangle sides with one endpoint P rotate and trace out the cones surface such that the cones bounding circle is orthogonal to the triangle side. In the former discussion, the equilibrium states of quarks and nucleons are bound to an inner dynamics of the system with different geometries added. The bounding genus 2,3 surfaces are interpreted as catastrophes with 2- and 6-roll mills. The flow for the 6-roll mill is for a quark-gluon plasma [1]. For quarks the POT field adds to them a unified potential without reference to general relativity GRE. In [8] is described that the gravitational field is obtained form the quark related projective 5-dimensional field 123456 as 4-dimensional projection 1256 while the electromagnetic interaction is projected to 1234 and scalar mass to 3456. As the example of ship boyancy shows, the gravitational action for nucleons can include a catastrophe angle β (replacing θ) GRE rescaling of mass. The lower parabola in figure 1 Has the upper cusp occuring and GRE adds a metrical scaling factor, known as Moebius transformation $MT \cos^2\beta = (r - R_s)/r$, r radius, R_s Schwarzschild radius of the ship. If the MT is normed to the coefficient matrix of order 6, it is the cross ratio for rotation 3. In generating octonians b doubling spacetime coordinates, 3 is bifurcated to 03, 0 as first octonian coordinate. Together in the octonian subspace 07 it is used as a G-compass (figure 3). An eigenvector of G presents color charges as a new force, independent of QCD, SI. Six valued, the color charges of 1,2,3,4,5,6 occur as charges on the segments-of the compass disk, generated when the compass needle turns with the sixth roots of unity.

For nucleons, the SI rotor states and dynamics allow to include for gravity in form of the tetrahedrons *rgb*-graviton field quantum that it adds in projection pr a stretching/squeezing of the nucleon triangle, related to the Schwarzschild radius of the nucleon. This is a metrical change like GRE, but done by the inner nucleon dynamics and pr. It has no interpretation as a spacetime curvature of its environment. The computation of the mass dependent scalar Rs is done according to Einstein and not repeated here. The reason for GRE is not spacetime curvature, but in dimensional reduced coordinates it is a stereographic, central, parametric catastrophe related projection of nucleons presented as a 2-dimensional bounding sphere S^2 into a tangent plane E with the radius-time differentials dr, dt as coordinates. They are rescaled by $dt' = \cos \beta \cdot dt$, keeping the area $drdt$ constant, the other rescaling is for $dr' = dr/(\cos \beta)$. Schwarzschild metric is $ds^2 = dr^2/(\cos^2 \beta) - \cos^2 \beta \cdot c^2 dt^2$. This is macroscopic used for GRE when stars are generated.

In a simplified model, in [4] is shown that the stretching/squeezing G-compass and *rgb*-gravitons effect for GRE can be shown by moving S^2 orthogonal up or downwards towards E and the shadow figure of a quark triangle gets larger or smaller in a pendulum motion. This is like a spring motion where a mass is attached at the end of the spring and released for an up/down directed oscillation. In addition the S^2 is rotating about the plane in a left- or reversed right-handed rotation in space.

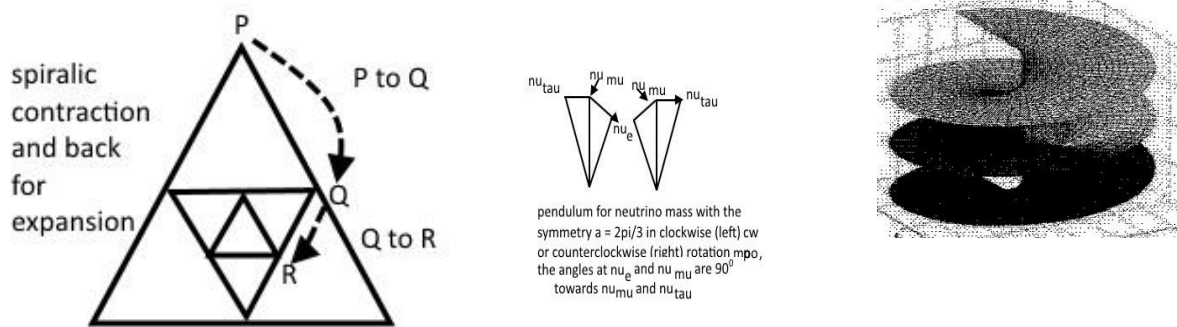


Figure 9 quarks chase one another like 3 dogs on a circle with constant speeds, an orthogonal, spiralic contraction of the the triangle area in proportion 1/2:1:2 occurs for the distance to E and reversely the up oriented oscillation is for stretching; neutrino oscillation is a similar (time dependent) pendulum motion; log(z) surface in case the effect is not reversed for expansion only or contraction, the cups are different and used all in up/down direction or down/up for length or other energy changes

The setting of the nucleons barycenter M is obtained in the SI rotor rotations as a D_3 presentation where the conic rotations in figure 8 left set barycentrical coordinates with intersection M. For the symmetry braking, the parabolic umbilic can be used. The MT as symmetry of a complex Riemannian sphere S^2 are projective obtained as quotients. In the former case, the linearized P^2 coordinates are $[(r-Rs), w=0, r]$ and norming this gives the MT $[(r-Rs)/r, 0, 1]$. For a planet P rotating about a sun Q with Rs as Schwarzschild radius of Q or for quarks rotating about M this means that the distance measure is unsymmetric with $|QP| = r$ and $|PQ| = r - Rs$. The parameter Rs is added as in a stereographic, central projection st. Recall that projective $st(x,y,z) = [-x, 0, z-1] = [1, 0, (z-1)/x]$ and set $z = r = x$ and scale 1 to Rs, $(z-1)/x = (r-Rs)/r$.

The parameter for pr is set differently from that in st in its catastrophe computation. For a cusp in the used parabolic umbilic the up/down area proportions 1/2:1:2 mean that the mass dependent potential is first on the maximal lower part of the cusp surface, jumps to the smaller middle part and up again to the minimal upper part potential. Light needs no EGR spacetime curvature. In its mirror effect, in braking its world line in a matter collision, it emits frequency. The inverse is for its world line breaking at a huge stars where it absorbes from Transformed mass frequency (blueshift) and shows double lensing. For the redshift is assumed that against a GR source at its projective infinity it has to break in time intervals its world line in an angle β and get a larger wave length. This revises the wave presentation of light which includes no world line braking for light waves. The angle β is spiralic as in figure 9 and increases wave length, it cannot change its speed. As catastrophe a cusp can be used transforming frequency to a relativistic mass potential. Time dependent mass changes on a world line are also observed for neutrino oscillations where the world line of the neutrino is not broken. As a substitute in this case it was assumed that a kg measuring base triple like a spin is added to a neutrino and changes in time like spin its three vectors with different kg mass weights attached such that observable is only one of them. This explanation is not useful for light since it is a pendulum motion as in figure 9 and redshift is not.

The light cusps are changing, enumerated for instance by winding numbers and changed for potentials use, starting always with the upper minimum level as in a complex logarithmic surface for the windings (figure 9). For the cubic roots

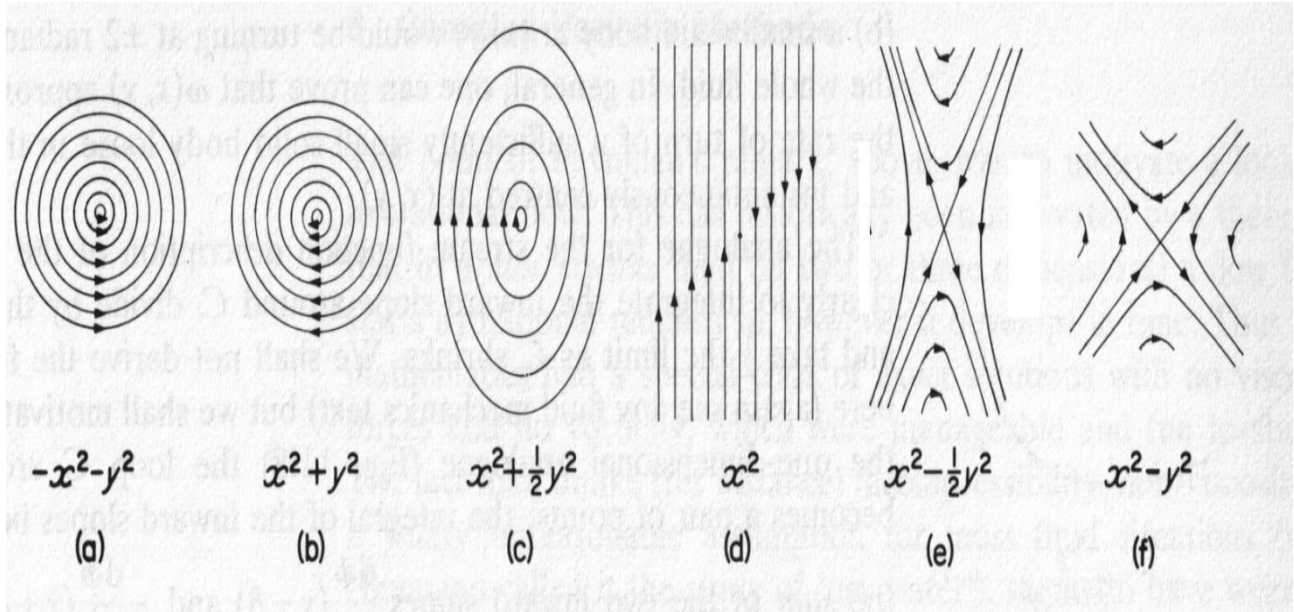
surface of a pendulum (standing wave) motion twice is a contraction (expansion), once no change and as reverse expansion (contraction) repeated in a time 6-cycle. The setting of orbital curves for systems in GR interaction is too long, the reader is referred to the references. An orbits plane angle towards a Minkowski double cone axis is important for this case and a rosette motion is due to adding a GR accelerated phase angle after one revolution in turning the larger Kepler ellipse diameter. For the affine Minkowski metric catastrophes are not needed. A critical Morse saddle (figure 8 right) can be applied. To metrical differentials is acting a Morse operator T on $(dr, cdt)T = (dr, -cdt)$, for the Eulidean metric rescaling in $ds^2 = \langle (dr, cdt)T, (dr, cdt) \rangle = dr^2 - c^2 dt^2$. Changing projective normal forms as quadric measures $\langle u, u \rangle$ require the application of a correlation and a projective transformation which associates with a point its dual hyperplane and the quadric occurs as those points incident with their hyperplane. For the Morse saddle, a circle as quadric is moved in the projective plane to a hyperbola which cuts the projective line g in two points while the circle has no point in common with g . A parabola has one point in common with g .

The 3- and 6-roll mill belong to nucleons, color charge, gluon quark flows and rolls and decays. For the case of a weak WI decay and weak bosons acts a 4 roll mill is (f) (as saddle and perturbed $(x^2+y^2)^2+a(x^2+y^2)$) in figure 10 for a lepton geometry [1] and symmetry braking, described by the $SU(2)$ Hopf map h and fiber twisted bundle. Either the electrical charge P is rotating with an angular speed on a latitude circle of S^2 (2-dimensional) or this circle is blown up by the fiber S^1 of the bundle to a torus location $S^1 \times S^1$ in $h^{-1}(S^2)$ where the point P is in S^3 as weak WI subspace in R^4 (spacetime) is a 45° leaning twisted circle in a plane towards the rotation axis of $h^{-1}(S^2)$ in direction of the space z -axis. For a 4 roll mill the two green, magenta rolls of SI have stopped, and the E(kin) roll 6 blue has to change its orientation. The rolls 145 red, yellow, turquoise are for electromagnetism. A neutral weak decay uses momentum 6 replacing 4 magnetic momentum, but the decay procedure is similar and not described here (see the references). The geometry needs a longer description. For a Heegard decomposition of S^3 into a two leptons decay (Feynman diagrams), a surface of genus n can be inserted in S^3 through a catastrophe. S^3 is bifurcated into two solid 3-dimensional parts having this surface as boundary. Time is added for their matter wave description which makes them 4-dimensional. The cut for the bifurcation can be along the S^3 equator (figure 5 left in lower 2 dimensions).

The case of solid balls is not for leptons having a torus surface of genus 1. An electrical or neutral charge is not bifurcated, one decay part has the electrical charge, the other one a neutral charge. The symmetry for the weak decay are the three Pauli matrices (with id added of order 4). The Hopf map retracts with them spacetime R^4 to space R^3 , using a twisted fiber S^1 for the projection of the unit sphere S^3 onto the unit sphere S^2 in R^3 . As well an electron as a neutron have then in reduced coordinates a S^2 spherical representation. The toroidal handle $S^1 \times S^1$ for them in S^3 is a fiber blow up of a latitude circle C in S^2 . On C a point charge of them is cw or mpo rotating for the - or + charge and neutrinos or antineutrinos. The weak decay generates in pairs an electron, antineutrino and a positron, neutrino. If in reduced coordinates of a fiber blow down their decay is demonstrated for a S^2 bifurcated at its equator, the electrons charged force vector sits e at the north pole of S^2 and the antineutrinos neutral charged force vector ν at the south pole of S^2 . The speed v of the ν momentum is higher than the second cosmic speed of W^- and ν can escape (bifurcating W^-) for the decay. The fact that weak bosons have as 3-dimensional systems a mass (set by a Higgs boson) is important. The structural instability of a weak boson is due to a missing perturbation of its the catastrophe potential. The elliptic umbilic for a SI 6roll mill has for the WI 4 roll mill as replacement a swallowtail catastrophe with $V_{abc} = x^5/5 + ax^3/3 + bx^2/2 + cx = 0$ where for instance $c = 0$ makes it unstable. Its derivative for a cusp catastrophe with $a/b < 0$ is structural stable. A 4 roll mill 1456 can also have $c \neq 0$ and is then structural stable, not decaying as flow (figure 10 (f)).

In this form it can describe a Gleason measuring spin-like base triple 145 for defining induction as angular momentum (substituted for 5) and real cross product of a magnetic momentum 4 belonging to an electrical currents 1 loop. The Hopf 2-dimensional geometry for the electron is S^2 , the electrical charged point 1 (eigenvector) rotates on a latitude circle of S^2 . The magnetic momentum eigenvector is in opposite direction in superposition with the spin as eigenrotation of the electron and sits with common endpoints at the north pole of S^2 . At the south pole sits the scalar mass as weight of the electron; the eigenrotation is clockwise cw. For a positron it is opposite mpo and also the magnetic momentum changes its direction and is parallel to spin. The Hopf S^2 geometry of an electron (also of a neutrino with neutral charge, momentum replacing electrical charge, magnetic momentum) is mapped up to S^3 for a field or flow like geometry. It is mapped down to the south pole tangent xy -plane of S^2 and presented as quotient $z = x + iy = z_2/z_1$, $z_1 = z + ict$ with $z_1 \neq 0$ and $z_1 = 0$ belongs to

the north pole of S^2 . For dihedrals D_n , $n = 2, 4, 6$ with n points on the circle replacing the 1 lepton or 3 quarks points in nucleons on $D_{1,3}$ a new method for getting stable solutions is: instead of rarely computable differential equations with boundary conditions solutions, a geometrical interpretation and inner flow dynamics is given. The number of poles on D_n uses the n th roots of unity as solutions of the polynomial equation $z^n - 1 = 0$. This is a cyclic Fibonacci-like sequence of a difference equation. The symmetries of order $2n$ apply to structural stable mills. For the octonian coordinate e_0 as tool a G-compass exists where G is a 2×2 -matrix (also as general relativistic metrical scaling) of order 6 belonging to the $E(\text{rot})$ cross ratio symmetry and the other n -roll mills have for $n = 2$ the $E(\text{magn})$ symmetry (Minkowski scaling T of Euclidean metric) of order 2, for $n = 4$ is the imaginary number i of order 4 belonging to the second Pauli matrix as conjugation responsible. For $n = 6$ it is the negative imaginary third root as sixth root of unity in the first quadrant. For $n = 8$ a possible root of this matrix with scalars including $\sqrt{2}$ and $1/\sqrt{2}$ can be chosen. It can be used for an octonian metric and symmetry



where the quaternionic case is repeated by doubling their spacetime coordinates. There is a D_n symmetry breaking since one part of matrices are D_3 elements for 123456 and G, T for 07 are not for D_4 or an extended D_8 but for the Einstein relativities and color charge cross ratios.

Figure 10 flows for potentials (a),(b),(c), for vector fields (d), 2 roll mill (e), 4 roll mill (f)

For a possible use of an 8 roll mill the reader is referred to the references and other articles of the author. The symmetry breaking for octonians is left as a research project. The use of S^2 as Riemannian sphere adds the Moebius transformations MT with cross ratios as invariants. Their actions are shown dynamically in the video [4]: allowed are numerical scalings of energy measures, translations, rotations of a system in spacetime and inversion $z \rightarrow 1/z$. This is for dark matter at the Schwarzschild radius as radius inversion and for speeds at the Minkowski light cone as $E(\text{kin})$ speed $v = \Delta x / \Delta t$ inversion for dark energy. There is also an oscillation inversion for rotational whirl 3 speed $\omega = d\phi/dt$ replacing v at an acoustic, heat 2 related Minkowski cone. Whirls are a third energy character [10] beside the duality between particle (momentum 6 or mass 5) and wave ($f = n$ for $U(1)$ 7) character.

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